The Rational Method*

David B. Thompson, Ph.D., P.E.
R.O. Anderson Engineering
Minden, Nevada

Draft: 20 January 2007

1. Introduction

For hydraulic designs on very small watersheds, a complete hydrograph of runoff is not always required. The maximum, or peak, of the hydrograph is sufficient for design of the structure in question. Therefore, the design discharge is the maximum value of the flood runoff hydrograph. A number of methods for estimating a design discharge have been developed. One such method was developed by Kuichling (1889) for estimating design discharge for small urban watersheds.

During the time since Kuichling's original development, the rational method became the basis for design of many small structures. In this context, small watershed refers to a watershed with a drainage area of a few tens of acres.[1] The rational method is described in most standard textbooks.[2]

[1] Texas Department of Transportation (TxDOT) experts suggest a maximum drainage area of 200 acres for rational method application. While many analysts consider this a "hard" limit, in actuality the limit really depends on watershed complexity. For a complex watershed (such as an urbanized watershed), this limit should probably be much less; for a rural watershed, the limit might be much larger. Therefore, it is the analyst's responsibility to determine applicability of the method and to justify application of the rational method based on professional judgment.

[2] See, for example, section 15.2 in Viessman and Lewis (2003).

2. Basics

The rational method is based on a simple formula that relates runoff-producing potential of the watershed, the average intensity of rainfall for a particular length of time (the time of concentration), and the watershed drainage area. The formula is

$$Q = C_u C i A, \tag{1}$$

where:

$$\begin{aligned} Q &= \text{design discharge } (\mathrm{L}^3/\mathrm{T}), \\ C_u &= \text{units conversion coefficient}, \\ C &= \text{runoff coefficient (dimensionless)}, \\ i &= \text{design rainfall intensity } (\mathrm{L}/\mathrm{T}), \text{ and} \\ A &= \text{watershed drainage area } (\mathrm{L}^2). \end{aligned}$$

The units conversion coefficient,[3] C_u, is necessary because the iA product, while it has units of L^3/T, is not a standard unit in the traditional units system.

3. Runoff Coefficient

The runoff coefficient, C, is a dimensionless ratio intended to indicate the amount of runoff generated by a watershed given a average intensity of precipitation for a storm. While it is implied by the rational method, equation 1, that intensity of runoff is proportional to intensity of rainfall, calibration of the runoff coefficient has almost always depended on comparing the total depth of runoff with the total depth of precipitation,

$$C = \frac{R}{P}, \tag{2}$$

where:

$$\begin{aligned} R &= \text{Total depth of runoff (L), and} \\ P &= \text{Total depth of precipitation (L).} \end{aligned}$$

The runoff coefficient represents the fraction of rainfall converted to runoff. Standard values are listed in table 1.

[3] The product of the dimensions of i, and A, is acre-inches per hour in traditional units. Dimensional analysis of this unit will show that this is equivalent to 1.00833 cubic feet per second. This is close enough to unity to be used as an equivalence for most cases.

Table 1: General runoff coefficients for the rational method. After Viessman and Lewis (2003).

Description	Runoff Coefficient
Business	
Downtown Areas	0.70–0.95
Neighborhood Areas	0.50–0.70
Residential	
Single-family	0.30–0.50
Multi-family detached	0.40–0.60
Multi-family attached	0.60–0.75
Residential suburban	0.25–0.40
Apartments	0.50–0.70
Parks, cemetaries	0.10–0.25
Playgrounds	0.20–0.35
Railroad yards	0.20–0.40
Unimproved areas	0.10–0.30
Drives and walks	0.75–0.85
Roofs	0.75–0.95
Streets	
Asphalt	0.70–0.95
Concrete	0.80–0.95
Brick	0.70–0.85
Lawns; sandy soils	
Flat, 2% slopes	0.05–0.10
Average, 2%–7% slopes	0.10–0.15
Steep, 7% slopes	0.15–0.20
Lawns; heavy soils	
Flat, 2% slopes	0.13–0.17
Average, 2%–7% slopes	0.18–0.22
Steep, 7% slopes	0.25–0.35

4. Storm Intensity

Storm intensity, i, is a function of geographic location, design exceedence frequency (or return interval), and storm duration.[4] It is true that the greater the return interval (hence, the lower the exceedence frequency), the greater the precipitation intensity for a given storm duration. Furthermore, as storm duration increases average precipitation intensity decreases.

The relation between these three components, storm duration, storm intensity, and storm return interval, is represented by a family of curves called the *intensity-duration-frequency* curves, or IDF curves. The IDF curves can be determined by analysis of storms for a particular site or by the use of standard meteorological atlases, such as TP-40 1963 and HYDRO-35 1977.[5]

[4] Actually, i is the *average* storm intensity for a storm with a duration equal to the time of concentration for the watershed.

[5] Although not included in this version of this paper, new precipitation data are being developed for IDF relations by National Oceanic and Atmospheric Agency.

Table 2: IDF parameters for Lubbock County.

	Return Interval (years)					
Parameter	2	5	10	25	50	100
e	0.830	0.821	0.813	0.816	0.808	0.810
b	47	60	69	82	88	101
d	10.0	10.1	10.1	10.1	10.1	10.0

For IDF curves, TxDOT[6] uses a formula for approximating the intensity-duration-frequency curve. The formula is

$$i = \frac{b}{(t_c + d)^e}, \tag{3}$$

where:

$$\begin{aligned} i &= \text{design rainfall intensity (in/hr)}, \\ t_c &= \text{time of concentration (min), and} \\ b, d, e &= \text{parameters.} \end{aligned}$$

For Lubbock County, the parameters are shown on table 2.

5. Time of Concentration

The time of concentration, t_c, of a watershed is often defined to be *the time required for a parcel of runoff to travel from the most hydraulically distant part of a watershed to the outlet.* It is not possible to point to a particular point on a watershed and say, "The time of concentration is measured from this point." Neither is it possible to measure the time of concentration. Instead, the concept of t_c is useful for describing the time response of a watershed to a driving impulse, namely that of watershed runoff.

In the context of the rational method, t_c represents the time at which all areas of the watershed that will contribute runoff to the watershed outlet are just contributing runoff to the outlet. That is, at t_c, the watershed is fully contributing. We choose to use this time to select the rainfall intensity for application of the rational method.

To elaborate, if storm duration is chosen to exceed t_c, then the rainfall intensity will be less than that at t_c. Therefore, the peak discharge estimated using the rational method

[6] TxDOT Hydraulic Design Guidelines, `http://manuals.dot.state.tx.us/dynaweb/colbridg/hyd` at the time of this writing.

will be less than the optimal value. Furthermore, if storm duration is chosen to be less than t_c, then the watershed is not fully contributing runoff to the outlet for that storm length, and the optimal value will not be realized, although a value for peak discharge will be computed that exceeds the value from the first case. Therefore, we choose the storm duration to be equal to t_c to estimate peak discharges using the rational method.

5.1. Estimating Time of Concentration

There are many methods for estimating t_c. In fact, just about every hydrologist or engineer has a favorite method. All methods for estimating t_c are empirical, that is, each is based on the analysis of one or more datasets. The methods in common use are not based on theoretical fluid mechanics.[7]

For application of the rational method, TxDOT recommends that t_c be less than 300 minutes (5 hours) and greater than 10 minutes. Other agencies require t_c to be greater than 5 minutes. The reason is that estimates of i become unacceptably large for durations less than 5 or 10 minutes. For long durations (such as longer than 300 minutes), the assumption of a relatively steady rainfall rate is less valid.

A number of methods are in common use for estimating time of concentration. For urban environment, Morgali and Linsley (1965) is sometimes used for planar flows. For rural environments, Kerby (1959) and Kirpich (1940) are useful for overland flow and channel flow, respectively.[8] The Natural Resources Conservation Service (NRCS) developed a method (U.S. Department of Agriculture, Natural Resources Conservation Service, 1986) that treats time of concentration (travel time) as having components related to overland flow (termed *sheet flow*), shallow-concentrated flow, and channel flow that are combined to produce an estimate of the time of concentration of a watershed. These methods are developed below.

5.2. Morgali and Linsley Method

For small urban areas with drainage areas less than ten or twenty acres, and for which the drainage is basically planar, the method developed by Morgali and Linsley (1965) is useful. It is expressed as

$$t_c = \frac{0.94(nL)^{0.6}}{i^{0.4}S^{0.3}}, \tag{4}$$

[7] The major point of this statement is that there is no *true value* for the time of concentration; all estimates are just that, estimates.

[8] Kerby (1959) could be useful for overland flow in urban environments too.

where:

$$
\begin{aligned}
t_c &= \text{time of concentration (min)},\\
i &= \text{design rainfall intensity (in/hr)},\\
n &= \text{Manning surface roughness (dimensionless)},\\
L &= \text{length of flow (ft), and}\\
S &= \text{slope of flow (dimensionless).}
\end{aligned}
$$

The Morgali and Linsley (1965) (equation 4) is implicit in that it cannot be solved for t_c without i. Therefore, iteration is required. Such a solution can be achieved by combining equation 3 with equation 4 and solving the result using a numerical method (such as a calculator solver). The solution of the two equations yields both t_c and i.[9]

5.3. Kirpich Method

For small drainage basins that are dominated by channel flow, Kirpich (1940) equation can be used. The Kirpich equation is

$$t_c = 0.0078(L^3/h)^{0.385} \tag{5}$$

where:

$$
\begin{aligned}
t_c &= \text{time of concentration (min)},\\
L &= \text{length of main channel (ft), and}\\
h &= \text{relief along main channel (ft).}
\end{aligned}
$$

Some authors use an adjustment factor for the Kirpich approach to correct for paved channels. The Kirpich method is limited to watersheds with a drainage area of about 200 acres.[10]

5.4. Kerby-Hatheway Method

For small watersheds where overland flow is an important component, but the assumptions inherent in the Morgali and Linsley approach are not appropriate, then the

[9] Of course, the method of successive substitution can be used with a graph of the IDF curve to arrive at a solution as well.

[10] Although Kirpich (1940) is applicable only to very small watersheds, in practice it is often used for watersheds with a single main stream (that is, a relatively simple structure) with drainage areas of 5 to 10 square miles.

Table 3: Kerby's roughness parameter.

Description	N
Pavement	0.02
Smooth, bare packed soil	0.10
Poor grass, cultivated row crops or moderately rough bare surfaces	0.20
Pasture, average grass	0.40
Deciduous forest	0.60
Dense grass, coniferous forest, or deciduous forest with deep litter	0.80

Kerby 1959 method can be used. The Kerby-Hatheway equation is

$$t_c = \left[\frac{0.67NL}{\sqrt{S}}\right]^{0.467} \tag{6}$$

where:

$t_c =$ time of concentration (min),
$N =$ Kerby roughness parameter (dimensionless), and
$S =$ overland flow slope (dimensionless).

Overland flow rarely occurs for distances exceeding 1200 feet. So, if the watershed length exceeds 1200 feet, then a combination of Kerby's equation and the Kirpich equation may be appropriate. Certainly, the combination of overland flow and channel t_c is an appropriate concept. Values for Kerby's roughness parameter, N, are presented on table 3.

5.5. NRCS Travel Time Method

The NRCS separates time of concentration (travel time) into three components: Sheet flow (overland flow), shallow concentrated flow, and channel flow. The overland flow component is computed using Overton and Meadows (1977),

$$t_c = \frac{25.2(nL)^{0.8}}{P_2^{0.5}S^{0.4}}, \tag{7}$$

Table 4: NRCS roughness parameter. After Engman (1986).

Description	n
Smooth surfaces (concrete, asphalt, gravel, or bare soil)	0.011
Fallow (no residue)	0.05
Cultivated soils, residue cover $\leq 20\%$	0.06
Cultivated soils, residue cover $> 20\%$	0.17
Short prairie grass	0.15
Dense grass	0.24
Bermudagrass	0.41
Range (natural)	0.13
Woods: Light underbrush	0.40
Woods: Dense underbrush	0.80

where:

$$
\begin{aligned}
t_c &= \text{time of concentration (min)},\\
n &= \text{Manning-like roughness coefficient (dimensionless)},\\
L &= \text{Overland flow length (ft)},\\
P_2 &= \text{2-year, 24-hour rainfall depth (in), and}\\
S &= \text{overland flow slope (dimensionless)}.
\end{aligned}
$$

The roughness parameter, n, is presented in table 4. Because depth of flow is relatively shallow for overland flow (sheet flow), values for the roughness parameter (a Manning-like parameter) are different than would be expected for application of Manning's equation to channel flows.

The second component of the NRCS approach is shallow-concentrated flow. For this component, NRCS (U.S. Department of Agriculture, Natural Resources Conservation Service, 1986) recommends use of Manning's equation to estimate flow velocity in one or more segments of the channel representing shallow, concentrated flow. Manning's equation is

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{8}$$

where:

$$\begin{aligned}
V &= \text{Mean velocity of flow (ft/s)},\\
n &= \text{Manning's roughness coefficient (dimensionless)},\\
R &= \text{Hydraulic radius } (A/P\text{; ft}), \text{ and}\\
S &= \text{Longitudinal slope (dimensionless)}.
\end{aligned}$$

In the NRCS WinTR-55 (WinTR-55 Workgroup, 2002)[11] model, two instances of Manning's equation are used to represent shallow-concetrated flow. They are

$$V = 16.1345S^{0.5}, \tag{9}$$

and

$$V = 20.3282S^{0.5}, \tag{10}$$

where:

$$\begin{aligned}
V &= \text{velocity (ft/s), and}\\
S &= \text{slope (dimensionless)}.
\end{aligned}$$

The time of concentration is then estimated by dividing the length over which shallow-concentrated flow occurs by the velocity,

$$t_c = \frac{L}{60V} \tag{11}$$

where t_c is in minutes.

As previously stated, both equation 9 and equation 10 are derived from Manning's equation (equation 8). In the case of equation 9, the hydraulic radius is assumed to be 0.4 feet with $n = 0.05$, and for equation 10, the hydraulic radius is assumed to be 0.2 feet and $n = 0.025$.[12] This means is other assumptions are possible. These estimates were convenient for NRCS computations and should be validated for the problem under consideration.[13]

The final component of the NRCS procedure is the channel travel time. In this case, they suggest the analyst apply Manning's equation (equation 8) to the channel to estimate mean velocity of flow. With that estimate, the time of concentration for channel flow is

$$t_c = \frac{L}{60V}, \tag{12}$$

[11] See `http://www.wcc.nrcs.usda.gov/hydro/hydro-tools-models-wintr55.html` at the time of this writing.

[12] These assumptions are presented in the WinTR-55 help file.

[13] In addition, the numerical precision of the constants for equations 10 and 9, as presented, is excessive. Three significant figures are sufficient.

where:

$$L = \text{Length of channel (feet), and}$$
$$V = \text{Mean velocity of flow from Manning's equation, and}$$
$$t_c = \text{Channel flow time of concentration (min).}$$

6. Putting It Together

More here later…

7. Perspectives

7.1. Time of Concentration

As mentioned previously in this article, there are as many ways of estimating time of concentration as there are hydrologists. Just the other evening, a young designer said "I was told that if you asked five hydrologists for the time of concentration for a drainage area, you'd get five answers — and none of them would be the same."

The young designer was correct — there is no true value. One of the reasons for this situation is that time of concentration is a concept and not an object with physical properties. That is, when engineers were searching for ways of estimating discharges for design purposes, it was recognized that time played a role in the size of the discharge. In other words, discharge scales not only with watershed drainage area, but with all of the elements in the watershed that go into the hydrograph, which would include watershed slope, main-stream slope, surface roughness (a distributed parameter), characteristic runoff intensity (rainfall excess intensity, another distributed parameter), and a host of others. When the rational method was developed, engineers had no tools for measuring, in some reasonable fashion, these properties.

So a simple parameter was needed. Those early analysts realized drainage area was a significant component of the hydrograph process. The height of the hydrograph scales with drainage area. But the length of the hydrograph and the height of the hydrograph also scale with the integrated response of the watershed to generated runoff. All of the influences on that response could not be estimated. So, a general lumped parameter of that response was created and it was the time of concentration.

As an aside, tools for assessing the characteristics of smaller and smaller plots are developing. Digital elevation models with a resolution of 30 meters are common. (Ten-meter resolutions are not far off.) Many analysts believe better (more accurate) results can come from more and more refined spatial analysis. This may be true, but the nature of those analyses must change from the relatively gross technologies generated 50 (or more) years ago to technologies that include the micro-dynamics of water motion at very small scales. It is this author's opinion that applying rainfall-runoff and runoff-motion technologies developed for watershed-scale processes to very small plots (ever decreasing in size) is tantamount to performing surgery with an axe.

7.2. Runoff Coefficient

The runoff coefficient, C, used in the rational method lumps all runoff-generating processes into a single parameter with one value. That a single value can represent all possible process outcomes is ludicrous. Some agencies attempt to correct that deficiency by providing a set of adjustment factors (sometimes based on exceedence probability) for the runoff coefficient. The tendency of analysts, then, is to apply the factors to runoff coefficients that are already inflated in an attempt to produce conservative results from a simple technology. As a result, many times the runoff coefficient applied to a problem approaches unity and the analyst might as well just use the average rainfall intensity for the time of concentration as the design discharge.

Although much work remains to be done, this author's opinion is that runoff coefficients published in standard texts are most likely conservative. For example, West (1998) executed a study of five playa lakes in Lubbock, Texas.[14] West (1998) observed the effective runoff coefficient from developed watersheds for storms occurring during his study to be about 0.3. The standard runoff coefficient for Lubbock, Texas is often about 0.7. This suggests a routine overdesign of over 100 percent.

7.3. Conservatism

This author has direct experience with over-conservatism. A natural tendency of designers is to work estimates always on the high (or conservative) side. This process is taught in engineering curricula either explicitly or implicitly and reinforced during the internship period of engineers. However, the drawback of such standard practice is that designs so created do not meet the risk level (exceedence probability) appropriate for

[14]The design guidelines for estimating design discharges is at `http://stormwater.ci.lubbock.tx.us/DrainageCriteriaManual/chapter_4.htm` at the time of this writing. (Note spaces in URL should be replaced with %20 or the URL may not work.)

the structure, but instead will pass events substantially greater than those required by local design codes.

There are several problems with this situation. First, the client (whether private or public) is paying for structures that are larger than required. Second, structures downstream not so designed may be impacted by flows exceeding their design flows. Third, problems that would be evidenced at the site may be moved downstream to other locations.

The risk level for a structure should be selected based on the outcome of structure failure. For a small culvert in a bar-ditch, the impact of an event that exceeds the capacity of the culvert is limited. However, failure of a similar culvert in another location might cause flooding of important structures and result in significant problems for a community.

It is this author's opinion that conservatism should not be applied at each step in the design process, but a rational (pun intended) decision be made by the designer to make the best estimate of the design discharge for an appropriate level of risk. Then, once the design discharge is estimated, a factor of safety can be applied during the structure-sizing process to ensure that errors (not blunders) in the design-discharge estimate are accommodated.

This approach must be taught by more experienced analysts to interns. It also must be implemented broadly throughout the design community. The intent is not to produce less expensive designs, although that is a spin-off of the process; the intent is to produce designs appropriate for the level of risk applied to a structure and agreed to by all parties — designers, owners, and regulators.

References

Engman, E. T. (1986). Roughness coefficients for routing surface runoff. *Journal of Irrigation and Drainage Engineering 112*(1), 39–53.

Kerby, W. S. (1959). Time of concentration for overland flow. *Civil Engineering 29*(3), 60.

Kirpich, Z. P. (1940). Time of concentration of small agricultural watersheds. *Civil Engineering 10*(6), 362. The original source for the Kirpich equation. (In PDF).

Kuichling, E. (1889). The relation between the rainfall and the discharge of sewers in populous districts. *Transactions, American Society of Civil Engineers 20*, 1–56.

Morgali, J. R. and R. K. Linsley (1965). Computer analysis of overland flow. *Journal of the Hydraulics Division 91*(HY3), 81–101.

National Oceanic and Atmospheric Administration (1977, June). Five- to 60-minute precipitation frequency for the eastern and central united states. Hydro-35. (In PDF).

Overton, D. E. and M. E. Meadows (1977). *Stormwater modeling*. Academic Press.

U.S. Department of Agriculture, Natural Resources Conservation Service (1986). Urban hydrology for small watersheds. Technical Release 55, U.S. Department of Agriculture.

U.S. Weather Bureau (1963). Rainfall frequency atlas of the united states for durations of 30 minutes to 24 hours and return periods from 1 to 100 years. Technical Report 40, U. S. Weather Bureau.

Viessman, W. and G. L. Lewis (2003). *Introduction to hydrology* (Fifth ed.). Prentice-Hall.

West, E. L. (1998). Hydrology of urban playa lakes in Lubbock, Texas. Master's thesis, Texas Tech University, Civil Engineering Department, Lubbock, Texas 79409.

WinTR-55 Workgroup (2002). WinTR-55 user manual. Technical report, Natural Resources Conservation Service.